Subject: Statistics for Economics
Course code: ECON4008
Topic: Mathematical Expectation
M.A. Economics (2nd Semester)

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Objectives

After studying this topic, you would be able to:

- find the expected values of random variables;
- understand the properties of expectation

Mathematical Expectation

- The probability of the happening of a the certain event is known as the probability of success (i.e. p) and the probability of non-happening of a certain even is known as the probability of failure (i.e. q). We always get p+q=1
- The mathematical expectation is the events which are either happening or non-happening a certain event in the experiment.
- Probability of an non-happening event is zero, which is possible only if the numerator is 0. Probability of happening of a certain event is 1 which is possible only if the numerator and denominator are equal.

Mathematical Expectation

- Mathematical expectation, also known as the expected value, which is the summation or integration of all possible values from a random variable.
- If x is a random variable which can assume any one of the values x_1, x_2, x_3, ….x_n with respective probabilities p_1, p_2, p_3, ….p_n then the mathematical expectation of x and denoted by E(x), is defined as:
- $E(x)= x_1 p_1+ x_2 p_2+….+ x_n p_n,$

  Where $\sum p_i = p_1+ p_2+…..+ p_n=1$

Mathematical Expectation

- If there is no occurrence of an event A, the mathematical expectation of an indicator variable will be 0, and if there is an occurrence of an event A, the mathematical expectation of an indicator variable will be 1.

- For example, a dice is thrown, the set of possible outcomes is { 1,2,3,4,5,6} and each of this outcome has the same probability with 1/6. Thus, the expected value of the experiment will be
{1(1/6)+2(1/6)+3(1/6)+4(1/6)+5(1/6)+6(1/6)}=21/6= 3.5.

Mathematical Expectation

- If x is a discrete random variable and f(x) is the value of its probability distribution at x, the expected value of x is

$$E(x) = \sum_{x} x.f(x)$$

- If x is a continuous random variable and f(x) is the value of its probability distribution at x, the expected value of X is

$$E(x) = \int_{-\infty}^{+\infty} x.f(x).dx$$

Properties of Mathematical Expectation

- $E(c) = c$, where c is a constant
- $E(cX) = c\,E(x)$, where c is a constant
- $E(aX+b)=aE(X)+b$, where a and b are constants
- **Addition rule of Mathematical expectation:** If X and Y are the two variables, then the mathematical expectation of the sum of the two variables is equal to the sum of the mathematical expectation of X and the mathematical expectation of Y.

  Or

  $E(X+Y)=E(X)+E(Y)$
- **Multiplication rule of Mathematical expectation:** The mathematical expectation of the product of the two random variables will be the product of the mathematical expectation of those two variables, In other words, the mathematical expectation of the product of the *n* number of independent random variables is equal to the product of the mathematical expectation of the *n* independent random variables

  Or

  $E(XY)=E(X)E(Y)$

Where X and Y are independent random variables

Properties of Mathematical Expectation

- The variance of a random variable is simple the expectation of its square minus the square of its expectation.
  $Var(X) = E[X - E[X]]^2 = E[X^2] - E[X]^2$

- if $E[X] = 0$ then $Var[X] = E[X^2]$.

- If a and b and are constants, then
  $Var[aX]=a^2Var[X]$
  $Var[X + b] = Var[X]$

- The covariance in terms of expectations of between two independent variables is:
  $Cov(X, Y) = E[(X - E[X])(Y - E[Y])]$.
  It can be shown that $Cov(X, Y) = E[XY]- E[X]E[Y]$.

Mathematical Expectation

Example 1:

- If you tosses two unbiased coins. You will win Rs 10 if 2 heads appear, Rs 5 if one head appears and Rs 2 if no head appears. Find the expected value of the amount won by you.
- **Solution:** In tossing two unbiased coins, the sample space, is S = {HH, HT, TH, TT}.
  P[2 heads] = 1/4 , P (one head) = 2/4 ,
  P(no head) = ¼
- Let x be the amount in rupees won by him. x can take the values 10, 5 and 2 with

  P[x = 10] = P(2heads) = 1/4

  P[x = 5] = P[1Head] = 2/4 , and

  P[x = 2] = P[no Head] = 1/4

Mathematical Expectation

Probability distribution of x is

x:	10	5	2
P(x):	1/4	2/4	1/4

Expected value of x is given as

$E(x)= x_1\, p_1+ x_2\, p_2+\ldots.+ x_n p_{n,}$

$E(x)=10(1/4)+5(2/4)+2(1/4)=10/4+10/4+2/4=22/4=5.5$

Thus, the expected value of amount won by you is Rs 5.5.

Mathematical Expectation

Example 2: For a continuous distribution whose probability density function is given by:

$$f(x) = \frac{3x}{4}(2-x), 0 \le x \le 2$$

find the expected value of X

Solution: Expected value of a continuous random variable X is given by.

$$E(x) = \int_{-\infty}^{+\infty} x.f(x).dx = \int_{0}^{2} x.\frac{3x}{4}(2-x).dx = \frac{3}{4}\int_{0}^{2} x^2(2-x).dx$$

$$= \frac{3}{4}\int_{0}^{2}(2x^2 - x^3).dx = \frac{3}{4}\left[2\frac{x^3}{3} - \frac{x^4}{4}\right]_0^2 = \frac{3}{4}\left[2\frac{2^3}{3} - \frac{2^4}{4} - 0\right]$$

$$= \frac{3}{4}\left[\frac{16}{3} - \frac{16}{4}\right] = \frac{3}{4}\left[\frac{16}{12}\right] = 1$$

Mathematical Expectation

Example 3: Given the following probability distribution:

X:	-2	-1	0	1	2
P(X):	0.15	0.30	0	0.30	0.25

Find (i) E(X), (ii) E(2X+3), and (iii) $E(X^2)$

Solution: (i) $E(X)= x_1\, p_1+ x_2\, p_2+....+ x_n p_{n,}$

$=(-2)(0.15)+(-1)(0.30)+(0)(0)+(1)(0.30)+(2)(0.25)$

$=-0.30-0.30+0+0.30+0.50=0.2$

(ii) $E(2X + 3) = 2E(X)+3=2(0.2)+3=0.4+3=3.4$

(iii) $E(X^2)= x_1^2\, p_1+ x_2^2\, p_2+....+ x_n^2 p_n$

$=(-2)^2(0.15)+(-1)^2(0.30)+(0)^2(0)+(1)^2(0.30)+(2)^2(0.25)$

$=(4)(0.15)+(1)(0.30)+(0)(0)+(1)(0.30)+(4)(0.25)$

$=0.6+0.30+0+0.30+1=2.2$

Mathematical Expectation

Example 4: A random variable x has the following probability distribution

x:	4	5	6	8
p:	0.1	0.3	0.4	0.2

Find $Var(X) = E[X - E[X]]^2$.

Solution: $Var(X) = E[X - E[X]]^2 = E[X^2] - E[X]^2$

$= \sum px^2 - (\sum px)^2$

x:	4	5	6	8	
p:	0.1	0.3	0.4	0.2	
px:	0.4	1.5	2.4	1.6	$\sum px=0.4+1.5+2.4+1.6=5.9$
px^2:	1.6	7.5	14.4	12.8	$\sum px^2=1.6+7.5+14.4+12.8=36.3$

$Var(X) = E[X - E[X]]^2 = \sum px^2 - (\sum px)^2 = 36.3 - (5.9)^2$

$=36.3-34.81=1.49$

Reference:

Gupta, S. C. (2015), *Fundamentals of Statistics*, Himalaya Publishing House.

Thank you..