Mahatma Gandhi Central University

ECON3020: Theory of Economic Growth

Course Code: ECON3020

Unit-iii, Growth Model with Exogenous Saving Rate

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The Solow-Swan Model

- The Solow-Swan model of economic growth postulates a continuous production function linking output to the inputs of capital and labour which leads to the steady state equilibrium of the economy.

- It is based on the following assumptions:

Assumptions

- 1) One composite commodity is produced.
- (2) Output is regarded *net* output after making allowance for the depreciation of capital.
- (3) There are constant returns to scale.
- (4) There are diminishing returns to an individual input.
- (5) The two factors of production—labour and capital—are paid according to their marginal physical productivities.

- **(6) Prices and wages are flexible.**
  (7) There is perpetual full employment of labour.
  (8) There is also full employment of the available stock of capital.
  (9) Labour and capital are substitutable for each other.
  (10) There is no technical progress.
  (11) The saving ratio is constant.
  (12) Saving equals investment.
  (13) Capital depreciates at the constant rate, *d*.
  (14) Population grows at a constant rate, *n*.

THE MODEL

- Given these assumptions, with unchanging technical progress, the production function is

$$Y = F(K,L)$$

where, *Y* is income or output, *K* is capital and *L* is labour.

- The condition of constant returns to scale implies that if we divide by *L*, the production function can be written as

$$\frac{Y}{X} = F(K/L, 1) = L.f(k) \ldots\ldots\ldots\ldots (1)$$

- **where, $Y = Y/L$ is output or income per worker, $k = K/L$ is the capital-labour ratio, and the function $f(k) = f(k, 1)$. Thus the production function can be expressed as**
- **$y=f(k)$.....(2)**

In the Solow-Swan model, saving is a constant fraction, *s*, of income. So saving per worker is *sy*. Since income equals output,

$sy=s.f(k)$.....(3)

The investment required to maintain capital per worker *k*, depends on population growth, and the depreciation rate, *d*.

Since it is assumed that population grows at a constant rate *n*, the capital stock grows at the rate *n*.*k* to provide capital to the growing population.

Since depreciation is a constant, *d*, per cent of the capital stock, *d*. *k* is the investment needed to replace worn-out capital.

This depreciation investment per worker *d*.*k* is added to *n*.*k*, the investment per worker to maintain capital-labour ratio for the growing population,

(nk+dk)=(n+d)k……(4)

- which is the investment required to maintain capital per worker.

  The net change in capital per worker (capita labour ratio) overtime is the excess of saving per worker over the required investment to maintain capital per worker,

  $$k=s.f(k)(n+d)k\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

- This is the fundamental equation for the Solow-Swan model, where the steady state corresponds to $k= 0$. The economy reaches a steady state when

  $$sf(k)=(n+d)k\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

- The Solow-Swan model is explained in Fig. 1.

- Output per worker y is measured along the vertical axis and capital per worker (capital-labour ratio), k, is measured along the horizontal axis.

- The $y = f(k)$ curve is the production function which shows that output per worker increases at a diminishing rate as k increases due to the law of diminishing returns.

- The $sf(k)$ curve represents saving per worker. The $(n + d)k$ is the investment requirement line from the origin with a positive slope equal to $(n+d)$.

- The steady state level of capital, is determined where the $sf(k)$ curve intersects the $(n+d)k$ line at point E.
- The steady state income is with output per worker P, as measured by point P on the production function $y = f(k)$.

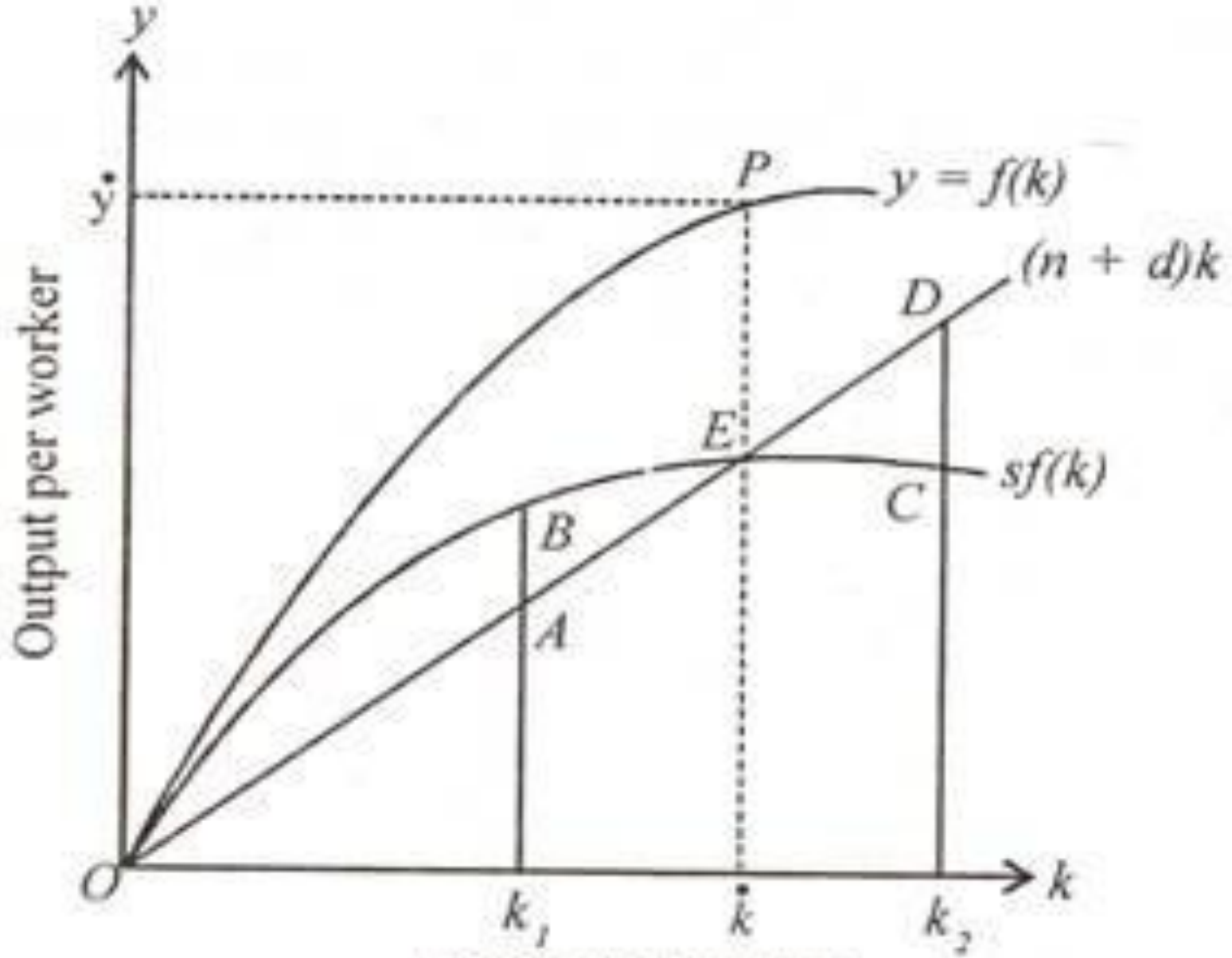
y
y*
P
y = f(k)
(n + d)k
D
E
sf(k)
C
B
A
O
k
k1
k*
k2
Output per worker
Capital per worker

Fig. 1

- **In order to understand why is a steady state situation, suppose the economy starts at the capital-labour ratio *k*1.**
- **Here saving per worker *k*1*B* exceeds the investment required to keep the capital-labour ratio constant, *k*1*A*, (*k*1*B* >*k*1*A*).**
- **Thus, *k* and *y* increase until k* is reached when the economy is in the steady state at point *E*.**
- ***Alternatively, if the capital-labour ratio is k2, the saving per worker, k2C, will be less than the investment required to keep the capital-labour ratios constant, k2D, (k2C < k2D). Thus y will fall as k falls to k* and the economy reaches the steady state E.***

The Solow-Swan model shows that the growth process is stable. No matter where the economy starts, forces exist that will push the economy over time to a steady state.

IMPLICATIONS OF THE MODEL

- There are some important implications or predictions of the Solow-Swan model of growth:
- 1. The growth rate of output in steady state is exogenous and is independent of the saving rate and technical progress.
- 2. If the saving rate increases, it increases the output per worker by increasing the capital per worker, but the growth rate of output is not affected.

Cont……..

- 3. Another implication of the model is that growth in per capita income can either be achieved by increased saving or reduced rate of population growth.
  This will hold if depreciation is allowed in the model.
- 4. Another prediction of the model is that in the absence of continuing improvements in technology, growth per worker must ultimately cease. This prediction follows from the assumption of diminishing returns to capital.

Thank You…..