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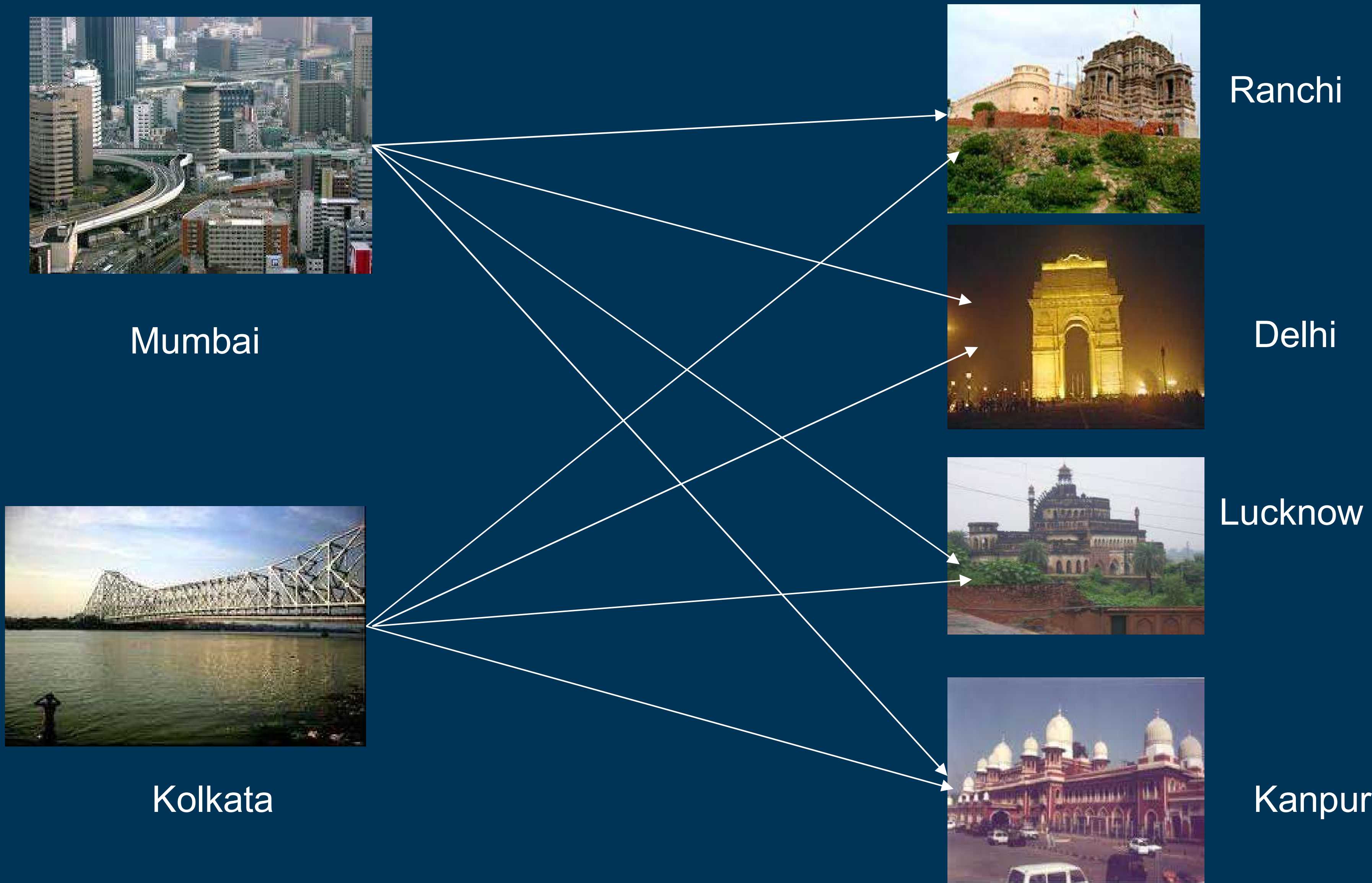
Mumbai
Kolkata
Ranchi
Delhi
Lucknow
Kanpur

Transportation problems

- The transportation problem is to transport various amounts of a single homogenous commodity that are initially stored at various sources / origins, to different destinations in such a way that the total transportation cost (or time) is minimum.

The Transportation Table

- The transportation problem is generally represented in a tabular form as shown below:

Destination

Origin

	D_1	D_2	. . . .	D_n
O_1	X_{11} C_{11}	X_{12} C_{12}	. . . .	X_{1n} C_{1n}
O_2	X_{21} C_{21}	X_{22} C_{22}	. . . .	X_{2n} C_{2n}
	. . . . .			
O_m	X_{m1} C_{m1}	X_{m2} C_{m2}	. . . .	X_{mn} C_{mn}

The mn large square are called the cells. The per unit cost of transporting from the i^{th} origin O_i to the j^{th} destination D_j is displayed in the lower right position of the cell. Any feasible solution to the Transportation problem is displayed in the table by variable X_{ij} at the upper left position of the $(i,j)^{th}$ cell. The various origin capacities and destination requirements are listed in the right most (outer) column and bottom (outer) row respectively. These are called rim requirements.

- **Feasible solution**: A set of non-negative individual allocations ($X_{ij} \geq 0$) which simultaneously removes deficiencies (wantage) is called a feasible solution.
- **Basic Feasible Solution**: A feasible solution to m-origin, n-destination problem is said to be basic if the number of positive allocation are m+n-1.
- **Optimal solution**: A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

Example: A company manufacturing air-coolers has two plants located at Mumbai and Kolkata with a weekly capacity of 200 units and 100 units, respectively. The company supplies air-coolers to its four show rooms situated at Ranchi, Delhi, Lucknow and Kanpur which have a demand of 70,100,100 and 30 units per week, respectively. The cost of transportation per unit (in Rs.) is shown in the following table:

	Ranchi	Delhi	Lucknow	Kanpur
Mumbai	90	90	100	100
Kolkata	50	70	130	85

Plan the production programme so as to minimize the total cost of transportation.

Solution: Following Vogel's Approximation method, the differences between the smallest and next-to-smallest costs in each row and each column are computed and displayed inside parenthesis (used in brackets) against the respective rows and columns. The largest of these differences is (40) and is associated with the first column of the transportation table. Since the minimum cost in the first column is C_{21}=50, we allocate X_{21} = min (100, 70) =70 in the cell (2, 1). This exhausts the requirement of the first column and, therefore, we cross off the first column.

	Ranchi	Delhi	Lucknow	Kanpur	Capacity	Row difference
Mumbai	90	90	100	100	200	(0)
Kolkata	70 50	70	130	85	100	(20)
Demand	70	100	100	30		
Column difference	(40)	(20)	(30)	(15)		

The row and column differences are now computed for the resulting reduced transportation table, the largest of these is (30) which is associated with the third column of the transportation table . Since the minimum cost in the third column is C_{13} =100, we allocate X_{13} = min (200, 100) =100 in the cell (1, 3). This exhausts the requirement of the third column and, therefore, we cross off the third column.

90	100 100	100	200	(0)
70	130	85	30	(20)
100	100	30		
(20)	(30)	(15)		

Continuing in this manner, the basic feasible solution shown in the following Table is obtained.

	Ranchi	Delhi	Lucknow	Kanpur	Capacity
Mumbai	90	70 90	100 100	30 100	200
Kolkata	70 50	30 70	130	85	100
Demand	70	100	100	30	

We now computed the number u_i(i=1,2) and v_j (j=1,2,3,4) using successively the equations $u_i + v_j = C_{ij}$ for all the occupied cells. For this, we arbitrarily assign u_i=0. Thus, we have

$$u_1 + v_2 = C_{12} \Rightarrow 0 + v_2 = 90 \Rightarrow v_2 = 90$$

$$u_1 + v_3 = C_{13} \Rightarrow 0 + v_3 = 100 \Rightarrow v_3 = 100$$

$$u_1 + v_4 = C_{14} \Rightarrow 0 + v_4 = 100 \Rightarrow v_4 = 100$$

$$u_2 + v_1 = C_{21} \Rightarrow -20 + v_1 = 70 \Rightarrow v_1 = 90$$

$$u_2 + v_2 = C_{22} \Rightarrow u_2 + 90 = 70 \Rightarrow u_2 = -20$$

The net evaluation for each of the unoccupied cells are now determined as follows:

$$Z_{11} - C_{11} = u_1 + v_1 - C_{11} = 0 + 90 - 90 = 0$$

$$Z_{23} - C_{23} = u_2 + v_3 - C_{23} = -20 + 100 - 130 = -50$$

$$Z_{24} - C_{24} = u_2 + v_4 - C_{24} = -20 + 100 - 85 = -5$$

Since we observe that all $Z_{ij} - C_{ij} \leq 0$, therefore, the optimum solution is reached.

The optimum solution is:

$$x_{12} = 70,\ \ x_{13} = 100,\ \ x_{14} = 30,\ \ x_{21} = 70,\ \ x_{22} = 30.$$

The associated cost with this solution is

$$Z = 70\times 90 + 100\times 100 + 30\times 100 + 70\times 50 + 30\times 70$$

$$= 6300 + 10000 + 3000 + 3500 + 2100$$

$$= Rs.23900$$

Reference Books:

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- F. S. Hiller and G. J. Leiberman, Introduction to Operations Research, McGraw-Hill International Edition, 1995.
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- S. D. Sharma, Operations Research, Kedar Nath and Ram Nath, 2012.

THANK YOU