Lecture-VII Programme: M. Sc./B.Sc. Physics

Wave Guide: Waveguide Resonators TM Mode

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- **If the cylindrical wave guide of definite length has its end faces then it forms what is known as resonator.**
- **Resonators are mainly utilized for the energy storage. At high frequencies ($\geq$ 0.1 GHz) the RLC circuit components are not sufficient when they utilized as resonators since the dimensions of the circuits are comparable to the operational wavelength, and thus, produce unwanted radiation.**
- **consequently, at high frequency range these RLC resonant circuits are replaced by electromagnetic cavity resonators.**

❖ **These resonator cavities are applied in <u>klystron tubes</u>, <u>band pass filters</u>, and <u>wave meters</u>. The microwave oven basically consists of a power supply, a waveguide feed, and an oven cavity. -**

❖**Suppose a rectangular cavity (or closed conducting box) of dimensions along X- axis is a, along Y- axis is b and along Z-axis is c and it is shown in Figure 1.**

❖ **We observe that the cavity is just a rectangular waveguide shorted at both ends. We thus expect to have standing wave and also TM and TE modes of wave propagation. This depends on how the cavity is excited, the wave can propagate in the *x*-, *y*-, or *z*-direction.**

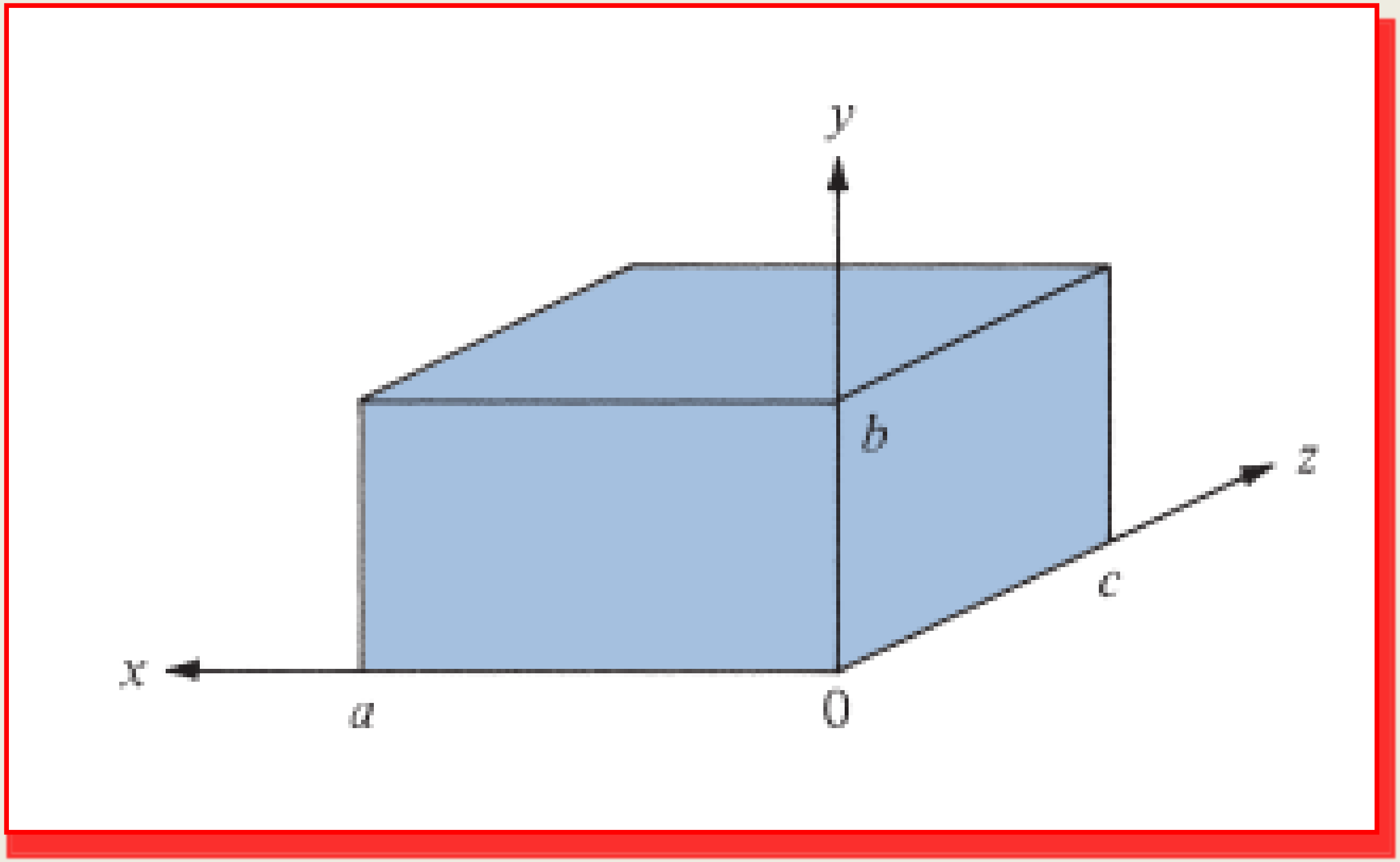

Figure 1: A rectangular resonator [*Ref-1] .

❖ **Here we will consider the positive *z*-direction as the "direction of wave propagation." In fact, there is no wave propagation.**

❖ **Somewhat, there are standing waves. As we know that a standing wave is a combination of two waves propagating in opposite directions.**

TM Mode to z

❖ **For propagation to z in transverse magnetic (TM mode) , for this case $H_z = 0$ and we let (As from previous lecture)**

$$\boxed{E_{zs}(x, y, z) = X(x)\, Y(y)\, Z(z)} \qquad \text{------------- [1]}$$

❖ It is the product of three solutions. We will follow the similar method taken in previous lecture –II for TEM mode calculation. So by using same method we get following solutions-

$$X(x) = c_1 \cos k_x x + c_2 \sin k_x x$$ ------------ [2]

$$Y(y) = c_3 \cos k_y y + c_4 \sin k_y y$$ ------------ [3]

$$Z(z) = c_5 \cos k_z z + c_6 \sin k_z z$$ ------------ [4]

where

$$k^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \varepsilon$$ ------------ [5]

❖ Now applying the following boundary conditions these are-

$$E_z = 0 \qquad \text{at} \quad x = 0, a \qquad \text{------------ [6]}$$

$$E_z = 0 \qquad \text{at} \quad y = 0, b \qquad \text{------------ [7]}$$

$$E_y = 0, E_x = 0 \qquad \text{at} \quad z = 0, c \qquad \text{------------ [8]}$$

❖ As like shown in previous lecture-III, the conditions in equations (6 and 7) are satisfied when $c_5 = 0$ and $c_3 = 0$ and thus we get-

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b} \qquad \text{------------ [9]}$$

where m = 1, 2, 3, . . . , n = 1, 2, 3, ... To raise the conditions in eq. (8), we notice that equation (23 in previous lecture -II) with ($H_{zs} = 0$) yields

$$\boxed{j\omega\varepsilon E_{xs} = \frac{1}{j\omega\mu}\left(\frac{\partial^2 E_{xs}}{\partial z^2} - \frac{\partial^2 E_{zs}}{\partial z\,\partial x}\right)} \qquad \text{-------------} \quad [10]$$

❖ Similarly, merging equations (17 and 20 from previous lecture-II) we get-

$$j\omega\varepsilon E_{ys} = \frac{1}{-j\omega\mu}\left(\frac{\partial^2 E_{zs}}{\partial y\,\partial z} - \frac{\partial^2 E_{ys}}{\partial z^2}\right)$$ ------------- [11]

From equations (10) and (11), it is obvious that equation (8) is satisfied only if-

$$\frac{\partial E_{zs}}{\partial z} = 0 \quad \text{at} \quad z = 0, c$$ ------------- [12]

This involves that $c_6 = 0$ and $\sin k_z c = 0 = \sin p\pi$. Thus,

$$\boxed{k_z = \frac{p\pi}{c}} \qquad \text{-------------} \quad [13]$$

where p = 0, 1, 2, 3, ... putting equations (9) and (13) into equations (2-4) which provide us the following solution-

$$\boxed{E_{zs} = E_o \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{c}\right)} \qquad \text{-------------} \quad [14]$$

where $E_o = c_2 c_4 c_5$. Other field components are obtained from equations (14) and equations (17 -22 from previous lecture-II).

❖ The phase constant β is obtained from equations (5), (9), and (13) as-

$$\boxed{\beta^2 = k^2 = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2 + \left[\frac{p\pi}{c}\right]^2}$$

------------ [15]

since

$$\boxed{\beta^2 = \omega^2 \mu \varepsilon}$$

------------ [16]

from eq. (15), we obtain the *resonant frequency* f_r

$$2\pi f_r = \omega_r = \frac{\beta}{\sqrt{\mu\varepsilon}} = \beta u'$$

$$f_r = \frac{u'}{2}\sqrt{\left[\frac{m}{a}\right]^2 + \left[\frac{n}{b}\right]^2 + \left[\frac{p}{c}\right]^2}$$

------------- [17]

❖ **From this equation it is clear that the following TM_{mnp} mode do not exist. These are as follow- TM_{000}, TM_{001}, TM_{010}, TM_{101}, TM_{011}.**

❖ **The physical possible lowest mode is of type TM_{110}.**

❖ **If two or more cavity modes have the similar resonant frequency, then the system is said to be degenerate and the resonant frequency is known as degenerate frequency. The order of the degeneracy is equal to the number of modes belonging to the degenerate frequency.**

❖ **From equation (17), we notice that the lowest-order TM mode is TM_{110}.**

❖**The corresponding resonant wavelength is-**

$$\boxed{\lambda_r = \frac{u'}{f_r} = \frac{2}{\sqrt{\left[\frac{m}{a}\right]^2 + \left[\frac{n}{b}\right]^2 + \left[\frac{p}{c}\right]^2}}} \qquad \text{-------------} \quad [18]$$

References:

1. Elements of Electromagnetics, 2nd edition by M N O Sadiku.
2. Engineering Electromagnetics by W H Hayt and J A Buck.
3. Elements of Electromagnetic Theory & Electrodynamics, Satya Prakash

- **For any query/ problem contact me on whatsapp group or mail on me**

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- **Next *** Wave Guide: Waveguide Resonators TE Mode, quality factor and Numerical based on topics**

Thank you