## Solution of Spherical, Cylindrical wave equation with Eigen value and function

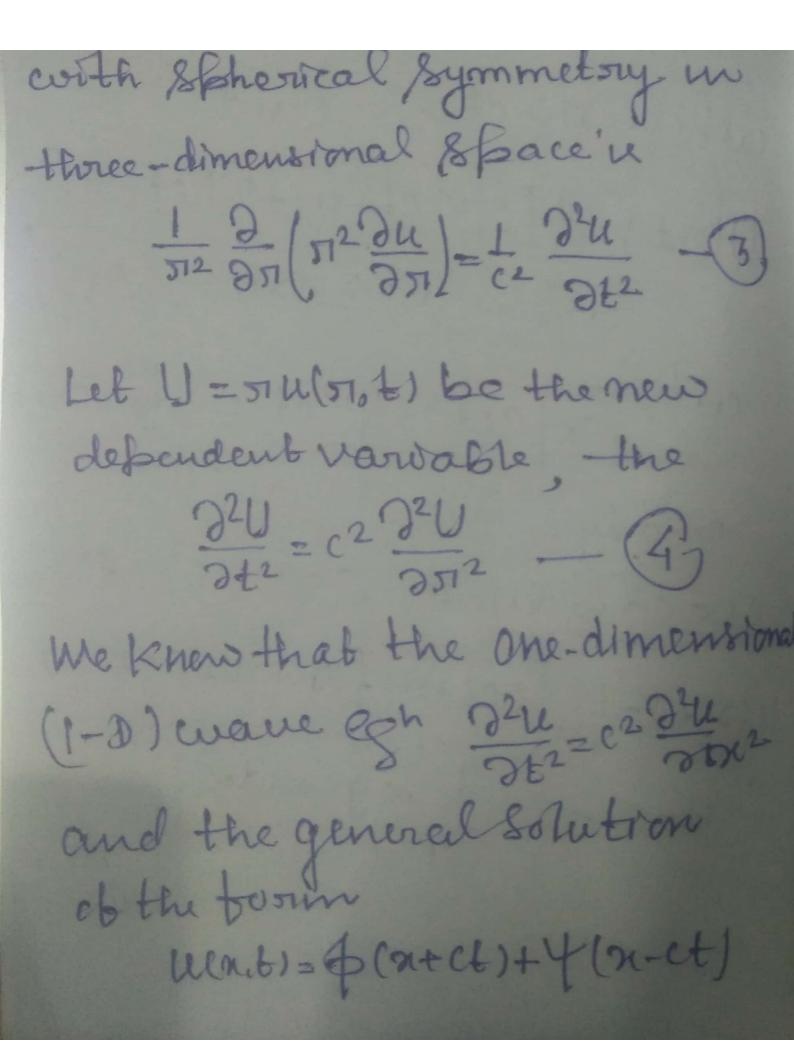


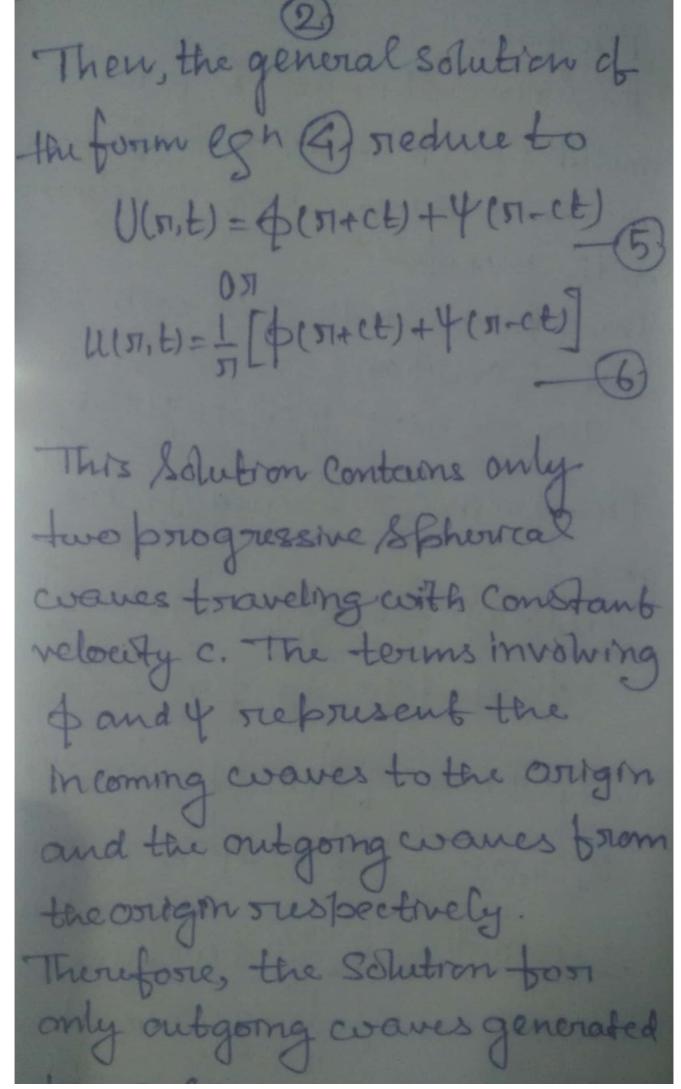
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Spherical coane equation The cuave egh 4+- c2 (Unn+lyg+lyzz)=0-(1) In Spherocal polar coordinate (51,0,6), x=110038 Sinf y=7 Sind Sinis Z = 51 Coz 6 the wave equation (1) takes the form 1 3 (212 0 1 22 0 0 0 0 (20) + 1 28 mg 20 (20) + 1 28 m20 20 - 1 2 2 L Solutions of the equation (2) are Called Ispherical Symmetric waves if u depends on si and tonly. Therefore the solubron 11=1111, t) which satisfies the wave equation with spherical symmetry in -three-dimensional & Bace's



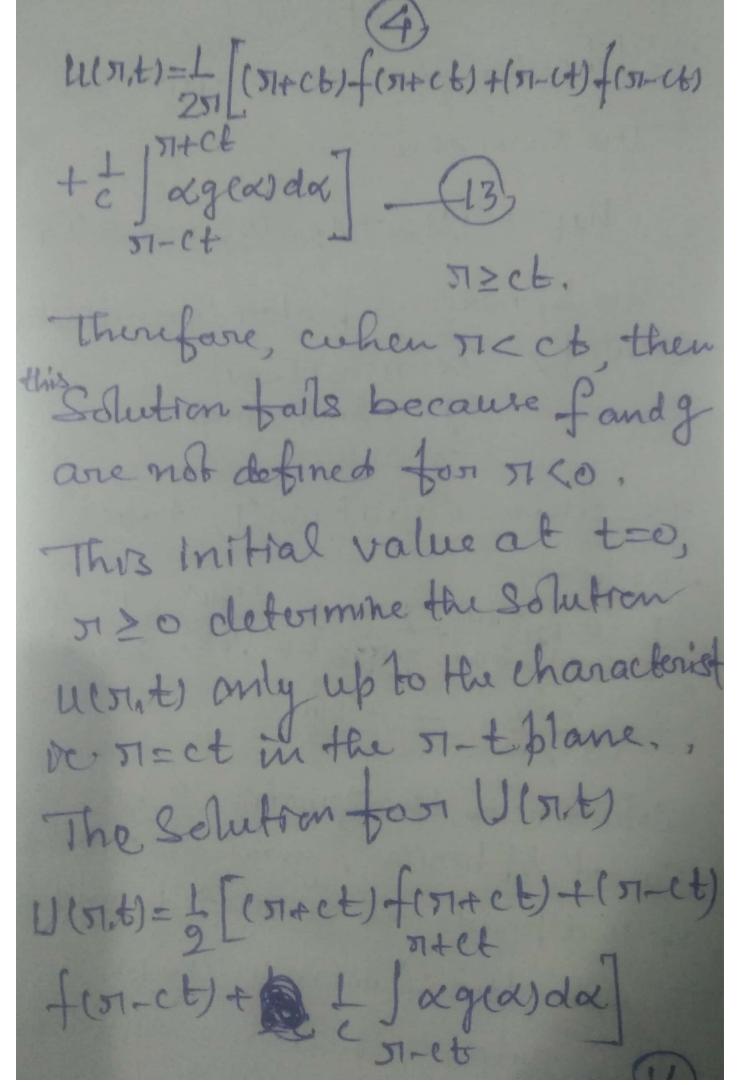


in coming waves to the onigin and the outgoing waves brom the onigen suspectively Therefore, the Solution for only outgoing waves generated by a source WN, b) = + 4 (n-(t) - (7) where 4 is to be determined from-the properties of the Source. In the care of thurs blows, u shows the velocity potential So that the limiting total blux through a sphere at constance couter at the only in Q(t) = lim 4x 512 Un (51,6) = lim 4x 112x {- 14(11-14)+14'] Quy = -4x4(-ct) -(8)

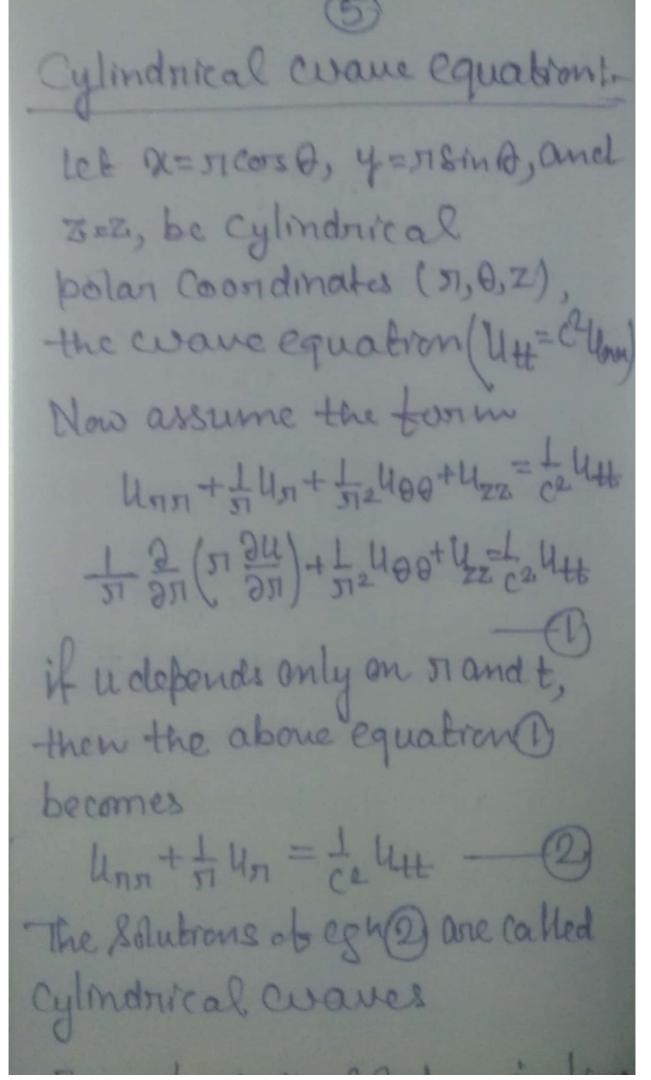
life that -The Source Strength Q is the multible on (product) de the Sunface area and normal Surface velocity do the monopole. The Solution of the IVP with initral condition u(1,0)=f(1), u(1,0)=g(1), 1120 Now brom egn 6) and 9), we い(21,0)=ナ(11)=ナ「中(11)+中(17) 中(カ)+中(の)= 打手(の) -(18) Ut(170も)= 「「毎(n7年)-4(n-cも) U+(17,0)= 号[中(17)-中(17)]=g(1) Integrating egn (1), we have

$$u_{+}(\pi,0) = \frac{1}{\pi} \left[ \frac{1}{\pi} (\pi) - \frac{1}{\pi} (\pi) \right] = g(\pi)$$
Integrating egn (1), we have
$$\frac{1}{\pi} (\pi) - \frac{1}{\pi} (\pi) = \frac{1}{\pi} \int_{-\pi}^{\pi} \alpha g(\alpha) d\alpha + k$$
Solving egn (10) and (2), waget
$$2 \frac{1}{\pi} (\pi) + \frac{1}{\pi} \int_{-\pi}^{\pi} \alpha g(\alpha) d\alpha + k$$

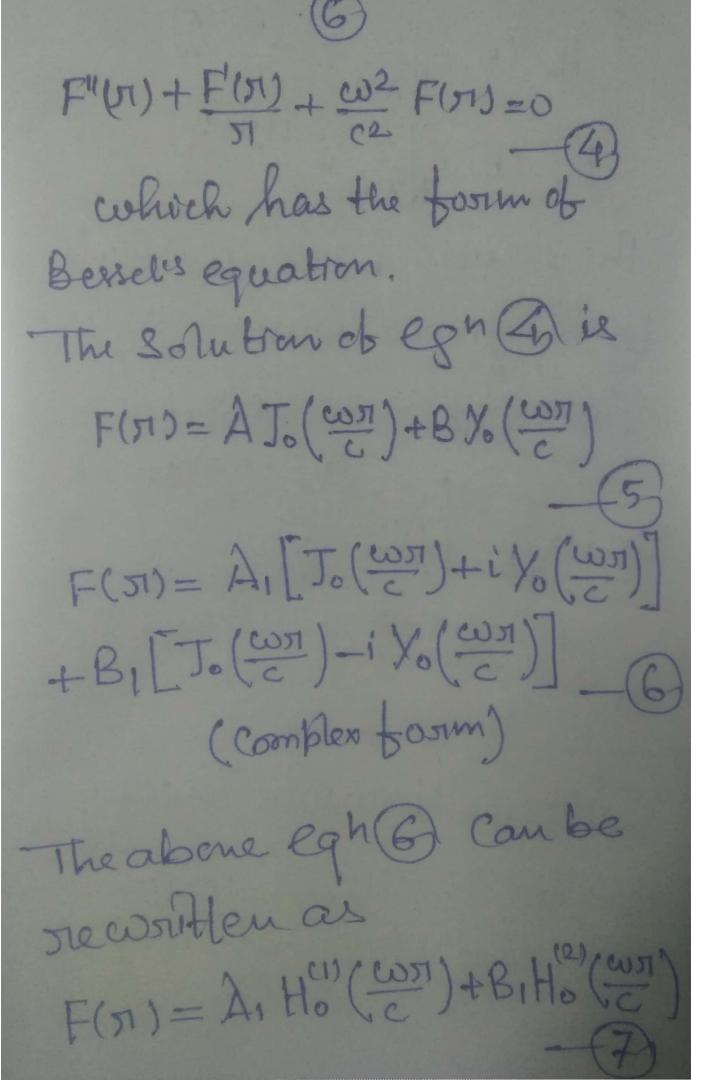
$$\frac{1}{\pi} \int_{-\pi}^{\pi} f(\pi) + \frac{1}{\pi} \int_{-\pi}^{\pi} \alpha g(\alpha) d\alpha + k$$
Similarly
$$\frac{1}{\pi} \int_{-\pi}^{\pi} f(\pi) + \frac{1}{\pi} \int_{-\pi}^{\pi} \alpha g(\alpha) d\alpha - k$$
Therefore,
$$u(\pi, b) = \frac{1}{2\pi} \left[ (\pi + cb) f(\pi + cb) + (\pi - cb) + (\pi - cb) + \frac{1}{\pi} (\pi + cb) + (\pi - cb) + \frac{1}{\pi} (\pi + cb) + \frac{1}{\pi} (\pi$$



U(51.6) = 1 [(51+ct) f(51+ct)+(51-ct) fin-ct)+ 2 L Jagrasda] when n2ct20 and when ct > 71 > 0 (15) where Acct)+4(ct)=0, for ct>0 Similarly eve can curre egn (15), then  $\pi(2)+32/(cc+2)+(cc+2)-(cc+2)$ f(4-11)+1 ] ct+11 ag(a)da]



becomes Unn+ ty Un = to lett The Solutrons ob egu(2) are called Cylindrical cuaves For a periodic Solution intime u(n,t)=Finseiwt (3) Then Qu = F'(n)eiwi 3º42 = - w2 Finseiws butting the abone o values, & 131 (22) = Cr 315 1 2 [IF'(I)e we] = - w2 Finze ITI.F'(n)e' + F(n) ne we + 4 (m) + w Finjelwt



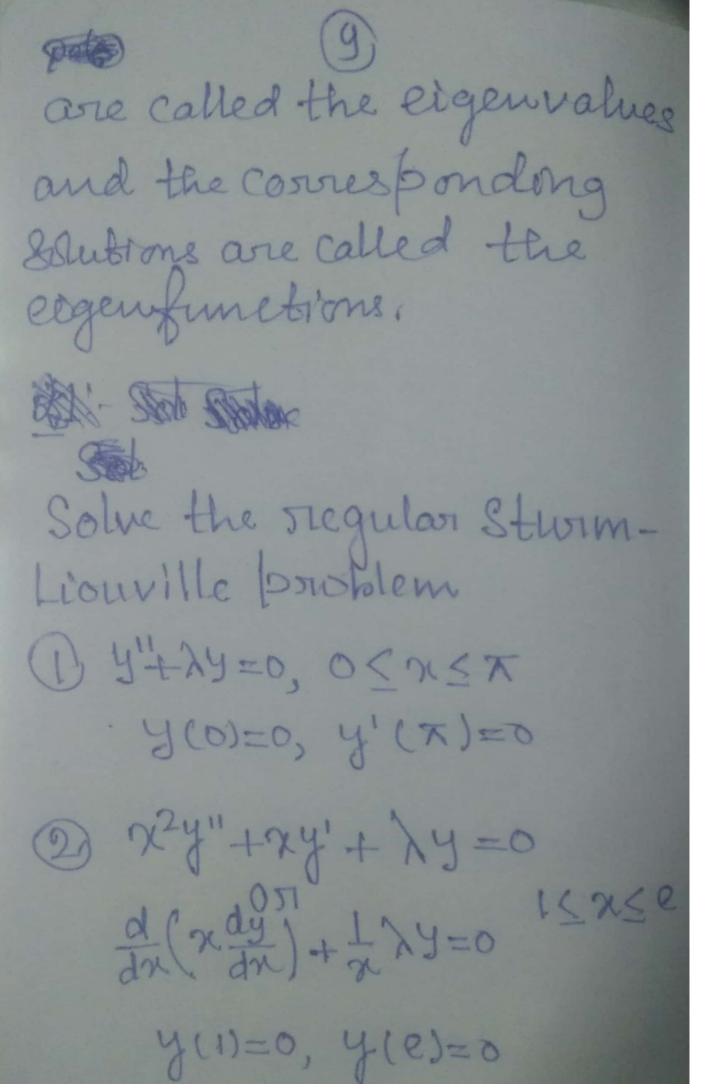
ne written as F(n) = A, Ho (wn) + B, Ho (wn) Ho= Jo (2)+i/o(2)-8 Ho = Jo (2) + i Vo (2) - 9 Ho, Ho are Hankel Junetrons defined by above expression egh (8) & (9). The Solution of 1-D wave ШП,t)=[A,Ho"(шП)+B,Ho"(шП))eiwb we know that asymptotic expression tosi Ho and 1902

(A) Ho (A) = [2] e (A-4) Ho (0) = 12 = i(0-2) The general perwodre Solution to the given wave egt in Cylindrical Coordinates Worth= 12c [A10 17 1 (2) (51+et) B1 e 4 e - i (2) (51-ct) Stwim-Liouville Theory Consider the Second-order differential equation of the

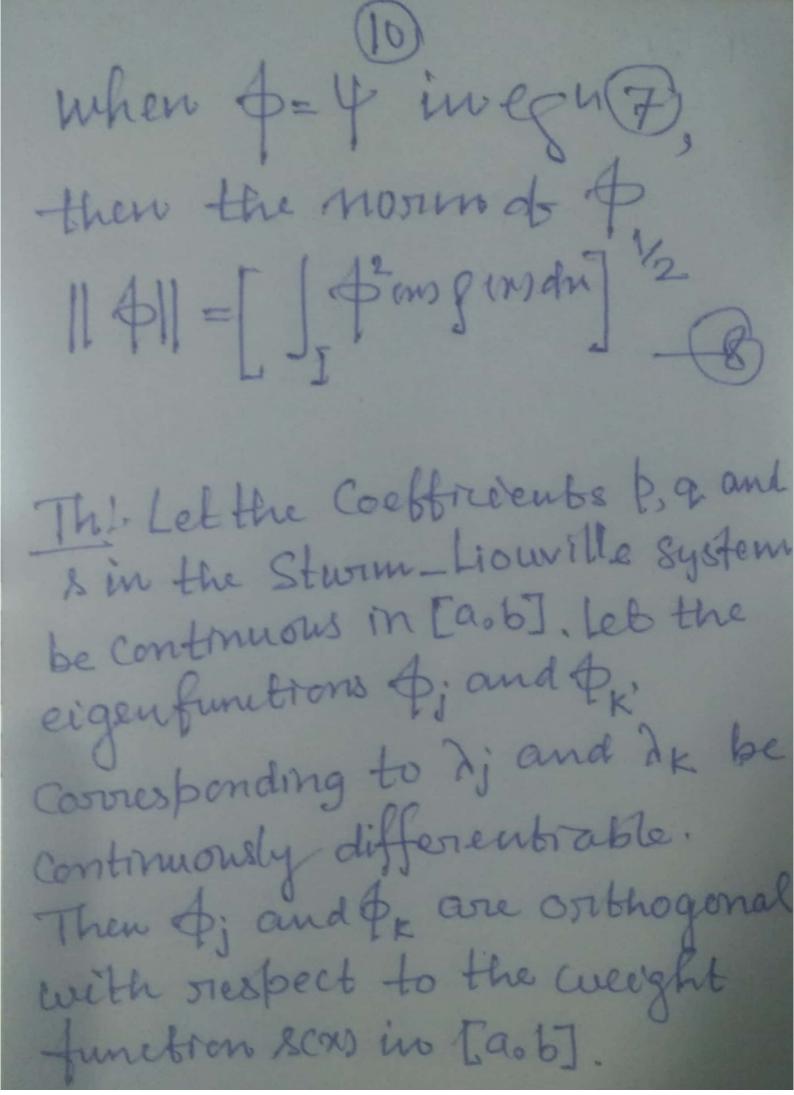
Stwim-Liouville Theory Consider the Second-Order differential equation of the a, (n) dy + a2(n) dy + [az(n)+) y=0 if pins= e auct) dt  $Q(n) = \frac{\alpha_0(n)}{\alpha_1(n)} \beta(n), \quad g(n) = \frac{\beta(n)}{\alpha_1(n)}$ d (pdy)+(9+2)4=0 which is known as the Sturm-Liouville equation cuhere de (p dn) +9

the above equ can be comitten as 1[y] + 1800y =0 -(4) where is a parameter independent of on, and p, 9 and & are neal valued function ob a. The Stwim-Liouville equation is called sugular in the interval [a.b] if the functions pens and sens core positive interval [asb]. For a given 2, there exist two L.I. Solutions of a nagular Stwim-Liouville

regular Stwim-Liouville equation in the interval [a.6]. New, eg (4) recorditing [[4] + \ sony=0, acx56 a,y(a) + a2y'(a) =0, ]-6 where as and as and likewise brand be are not seal numbers, is called a regular Stwim-Liouville The values of & for wehich the Stwim- Liouville System has a montorivial Solution



り"ナカリ=0,一下三九三大 y(-T)=y(T), y'(-T)=y'(T) Let pens and 4 (m) be any realvalued integrable functions on an interval I. Then & and 4 are savd to be orthogenal on] tunction pcasso, iff my funguations The interval I may be of infinite On it may be either open on closed at one on both ends of the



eigenfunctions \$; and \$, Cornesponding to 2 and 2k be continuously differentiable. Then \$ and \$ are onthogonal with nespect to the weight function sex ino [a.6]. Th: All the eigenvalues of a negular Stum-Liouville System with sers >0 are real. This If from and from are any -two Solutions of the equation [[4] + \ 100) y=0 on [a,b]. then bons W(9, 9) m) = constant, where W is the Wnonskian. This An eggenfunction of a regular Shurm- Liouville is unique except for a Constant factor.