Lecture-XIII Programme: M. Sc. Physics

RADIATION: MAGNETIC DIPOLE RADIATION

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Magnetic Vector Potential

- In this lecture we will focus on radiation due to magnetic dipole. In this regards first we will define magnetic vector potential due to current carrying loop. Thereafter we will define electric and magnetic fields for this system.
- Consider now that we have a wire loop of radius *b* (Figure 1), around which we drive an alternating current I, the expression for this alternating current is defined as :

$$I(t) = I_0 \cos(\omega t). \quad \text{------------- [1]}$$

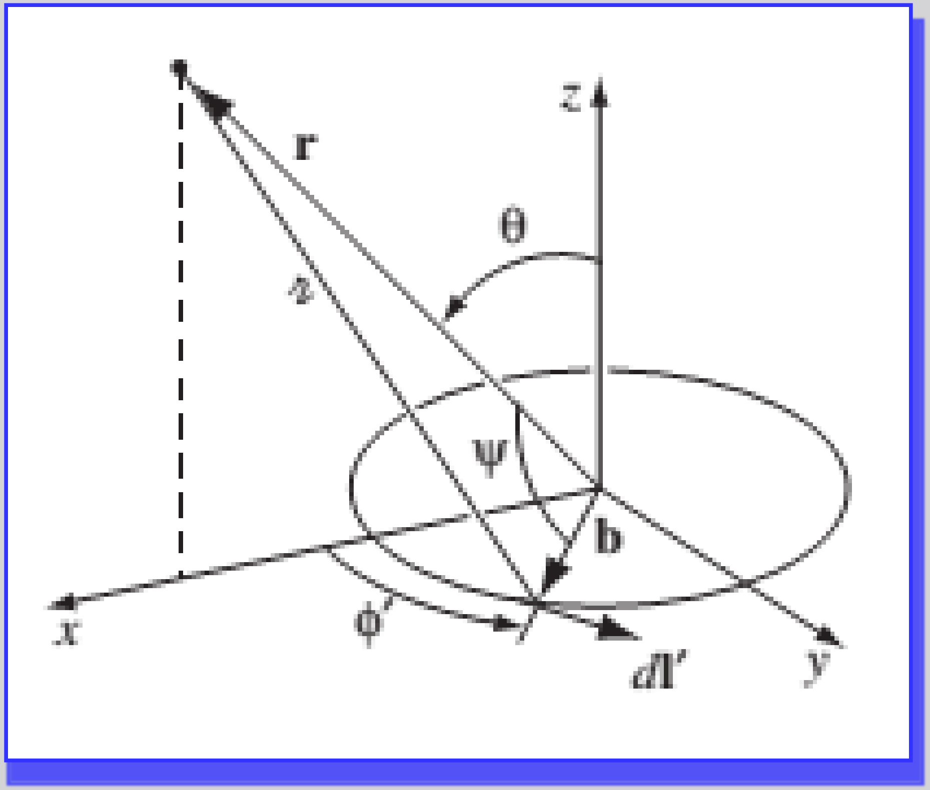

FIGURE 1: Representation of current loop and point where the vector potential A. ** [REF-1]

❖ This is a model for an oscillating *magnetic dipole*, therefore the expression for the magnetization *m(t)*:

$$\mathbf{m}(t) = \pi b^2 I(t)\,\hat{\mathbf{z}} = m_0 \cos(\omega t)\,\hat{\mathbf{z}}, \qquad \text{------------- [2]}$$

where

$$m_0 \equiv \pi b^2 I_0 \qquad \text{------------- [3]}$$

❖ As the loop is uncharged, therefore the scalar potential is zero.

❖ Expression for the retarded vector potential is:

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{I_0 \cos[\omega(t - \mathscr{r}/c)]}{\mathscr{r}} \, d\mathbf{l}'.$$ ------------- [4]

❖ For a point r directly over the *x* axis (Fig. 1), vector potential, A must intend in the *y* direction, because the *x* components from symmetrically located points on either side of the *x* axis will cancel. Hence-

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0 I_0 b}{4\pi} \, \hat{\mathbf{y}} \int_0^{2\pi} \frac{\cos[\omega(t - \mathscr{r}/c)]}{\mathscr{r}} \cos\phi' \, d\phi'$$ ------------- [5]

(cos φ serves to choose out the *y*-component of *d*l). From the law of cosines,

$$\mathscr{r} = \sqrt{r^2 + b^2 - 2rb\cos\psi}\,, \qquad \text{------------ [6]}$$

where ψ is the angle between the vectors r and b:

$$\mathbf{r} = r\sin\theta\,\hat{\mathbf{x}} + r\cos\theta\,\hat{\mathbf{z}}, \quad \mathbf{b} = b\cos\phi'\,\hat{\mathbf{x}} + b\sin\phi'\,\hat{\mathbf{y}}. \qquad \text{------------ [7]}$$

So $rb\cos\psi = \mathbf{r}\cdot\mathbf{b} = rb\sin\theta\cos\varphi$, and thus,

$$\mathscr{r} = \sqrt{r^2 + b^2 - 2rb\sin\theta\cos\phi'}. \qquad \text{------------ [8]}$$

For a "perfect" dipole, we want the loop to be extremely small:

So $rb \cos \psi = \mathbf{r} \cdot \mathbf{b} = rb \sin \theta \cos \varphi$, and thus,

$$\imath = \sqrt{r^2 + b^2 - 2rb \sin\theta \cos\phi'}.$$

------------ [8]

- For a "perfect" dipole, we desire the loop to be very small:

Approximation 1 : $b \ll r$

To first order in b, then,

$$\imath \cong r\left(1 - \frac{b}{r} \sin\theta \cos\phi'\right)$$

therefore

$$\frac{1}{\imath} \cong \frac{1}{r}\left(1 + \frac{b}{r}\sin\theta\cos\phi'\right)$$ -------------- [9]

and

$$\cos[\omega(t - \imath/c)] \cong \cos\left[\omega(t - r/c) + \frac{\omega b}{c}\sin\theta\cos\phi'\right]$$

$$= \cos[\omega(t - r/c)]\cos\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right) - \sin[\omega(t - r/c)]\sin\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right)$$

-------------- [10]

❖ As previous to, we also suppose the size of the dipole is small compared to the wavelength radiated:

Approximation 2 : *b << c/ω*

In that situation,

$$\cos[\omega(t - \mathscr{r}/c)] \cong \cos[\omega(t - r/c)] - \frac{\omega b}{c} \sin\theta \cos\phi' \sin[\omega(t - r/c)].$$

-------------- [11]

putting Equations 9 and 10 into Eq. 5, and leaving the second-order term:

$$\mathbf{A}(\mathbf{r}, t) \cong \frac{\mu_0 I_0 b}{4\pi r} \hat{\mathbf{y}} \int_0^{2\pi} \Big\{ \cos[\omega(t - r/c)] + \; b \sin\theta \cos\phi' \times \left(\frac{1}{r} \cos[\omega(t - r/c)] - \frac{\omega}{c} \sin[\omega(t - r/c)] \right) \Big\} \cos\phi' \, d\phi'.$$

----------- [12]

- The first term integrates to zero:

$$\int_0^{2\pi} \cos\phi' \, d\phi' = 0.$$

- The second term engages the integral of cosine squared:

$$\int_0^{2\pi} \cos^2\phi' \, d\phi' = \pi.$$

Inserting this in, and noting that in general A points in the $\hat{\boldsymbol{\phi}}$ -direction, At last we conclude that the vector potential of an oscillating perfect magnetic dipole is-

$$\mathbf{A}(r,\theta,t) = \frac{\mu_0 m_0}{4\pi}\left(\frac{\sin\theta}{r}\right)\left\{\frac{1}{r}\cos[\omega(t-r/c)] - \frac{\omega}{c}\sin[\omega(t-r/c)]\right\}\hat{\boldsymbol{\phi}}.$$

------------- [13]

❖ For the static case the limit (ω = 0), we get the well-known relation for the potential due to a magnetic dipole-

$$\mathbf{A}(r,\theta) = \frac{\mu_0}{4\pi}\frac{m_0 \sin\theta}{r^2}\hat{\boldsymbol{\phi}}. \qquad \text{------------- [14]}$$

❖ In the radiation zone,

Approximation 3 : $r >> c/\omega$

the first term in A is small, so neglecting it

$$\mathbf{A}(r,\theta,t) = -\frac{\mu_0 m_0 \omega}{4\pi c}\left(\frac{\sin\theta}{r}\right)\sin[\omega(t - r/c)]\,\hat{\boldsymbol{\phi}}. \qquad \text{------------- [15]}$$

Expressions for the Electric and Magnetic Field

- To find the Poynting vector we get the fields at large *r* from Potential A:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} = \frac{\mu_0 m_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\,\hat{\boldsymbol{\phi}}, \quad \text{-------------- [16]}$$

and

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{\mu_0 m_0 \omega^2}{4\pi c^2}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\,\hat{\boldsymbol{\theta}}. \quad \text{-------------- [17]}$$

- We applied approximation 3 in finding B. These fields are in phase, mutually perpendicular, and transverse to the direction of propagation $(\hat{\mathbf{r}})$.
- The ratio of their amplitudes is $E_0/B_0 = c$, all of which is as likely for electromagnetic waves.

❖ They are, in fact, really same in structure to the fields of an *oscillating electric* dipole (in previous lecture), only difference is that B pointed in the $\hat{\theta}$ direction and E in the $\hat{\phi}$ direction, while for electric dipoles it is the other way around.

Poynting Vector and Total Radiated Power

❖ The energy flux for magnetic dipole radiation is

$$\mathbf{S}(\mathbf{r}, t) = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}) = \frac{\mu_0}{c}\left\{\frac{m_0\omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\right\}^2 \hat{\mathbf{r}},$$

------------- [18]

the intensity is

$$\langle \mathbf{S} \rangle = \left(\frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3}\right)\frac{\sin^2\theta}{r^2}\hat{\mathbf{r}},$$

------------- [19]

and the total radiated power is

$$\langle P \rangle = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}. \qquad \text{-------------- [20]}$$

❖Once again, the intensity profile has the shape of a donut (Fig. 3), and the power radiated varies like ω^4.

❖There is, yet, one key difference between electric and magnetic dipole radiation: For arrangements with comparable dimensions, the power radiated electrically is extremely greater.

❖Comparing equation (24) from previous lecture-XII and equation (20),

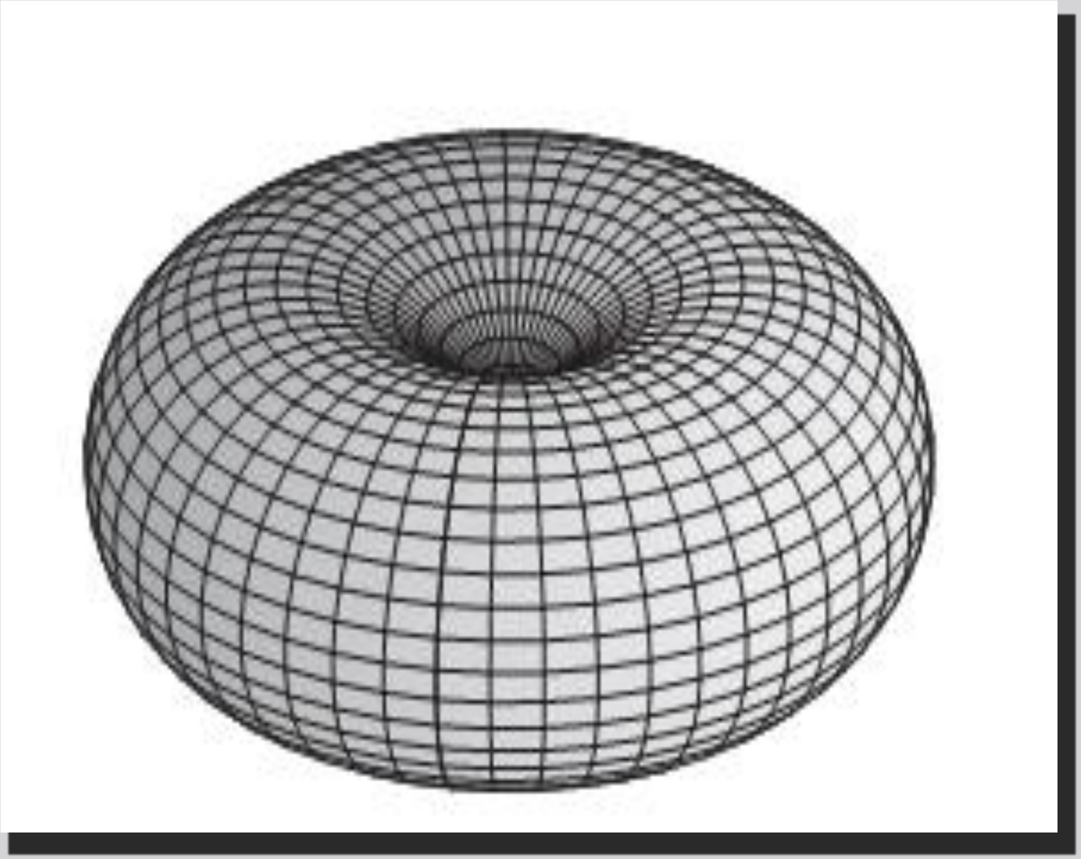

FIGURE 2: Representation of radiated power profile due to a magnetic dipole through a sphere of radius r . **[REF-1]

$$\frac{P_{\text{magnetic}}}{P_{\text{electric}}} = \left(\frac{m_0}{p_0 c}\right)^2, \quad \text{-------------- [21]}$$

Note: where $m_0 = \pi b^2 I_0$, and $p_0 = q_0 d$. The amplitude of the current in the electrical case was $I_0 = q_0\omega$. Setting $d = \pi b$, for the sake of comparison, we obtain:

$$\frac{P_{\text{magnetic}}}{P_{\text{electric}}} = \left(\frac{\omega b}{c}\right)^2. \quad \text{-------------- [22]}$$

❖ **However *ωb/c* is exactly the quantity we supposed was much small (approximation 2), and here it shows *squared*. Usually, then, one should suppose electric dipole radiation to dominate. Only when the system is watchfully contrived to prohibit any electric contribution (as in the case just treated) will the magnetic dipole radiation make known itself.**

Numerical problems

1. Calculate the electric and magnetic fields of an oscillating magnetic dipole without using approximation 3. [Do they look familiar? Compare Prob. 9.35.] Find the Poynting vector, and show that the intensity of the radiation is exactly the same as we got using approximation 3

2. Treating electric dipole to be equivalent to an accelerated charge, calculate:

(i) The dipole moment amplitude p_0 in terms of charge q and acceleration a of the accelerated charge.

(ii) Instantaneous rate of radiation from the charge.

References:

1. *Introduction to Electrodynamics , David J. Griffiths*

2. *Modern Electrodynamics, Andrew Zangwill*

3. *Elements of Electromagnetics, 2nd edition by M N O Sadiku*

4. *Engineering Electromagnetics by W H Hayt and J A Buck.*

5. *Elements of Electromagnetic Theory & Electrodynamics, Satya Prakash*

- **For any query/ problem contact me on whatsapp group or mail on me**

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- **Next *** we will discuss the Radiation from an Arbitrary Source**

  (Larmor formula) and numerical problems.

Thank you